Abstract

Quantum entanglement is recognized as one of the most important resources in the rapidly expanding field of quantum information theory. Great progress has been made toward the detection and generation of quantum entanglement experimentally. However, there are still many open problems on theory of quantum entanglement for both bipartite and multipartite quantum states. In this thesis, we mainly study two important problems in the theory of quantum entanglement: the separability problem and the local unitary (LU) equivalence of quantum states.

One part of this thesis concerns the fundamental problem of separability of quantum states. Firstly, we study the positive partial transposition (PPT) condition and construct the explicit separable pure state decomposition for a set of bipartite quantum states satisfying the PPT conditions. Secondly, for arbitrary dimensional bipartite ($m \otimes n$) mixed states, we propose an inequality for detecting entanglement. This inequality gives a necessary condition for separability of mixed states, namely, any violation of the inequality implies entanglement. It is shown that our inequality can detect entanglement of considerable states such as the Horodecki's state, isotropic state. Thirdly, we generalize the approach to deal with the separability of multipartite states, in particular, to distinguish different kinds of entanglement in three-qubit systems. We first express the bipartite entanglement of a three-qubit pure state in terms of the expectation values of Pauli operators. Based on this, we derive some inequalities which can be viewed as entanglement witnesses to detect the separability of three-qubit pure states completely. For the entanglement detection of three-qubit mixed states, we propose two kinds of inequalities in terms of the expectation values of complementary observables.

In the another part, we use various approaches to study the local unitary equivalence problems of quantum states. One fact is that two entangled states are said to be equivalent in implementing the same quantum information task if they can be obtained with certainty from each other by means of locally operations and classical communicate (LOCC). Theoretically, two pure states in the same equivalent class if and only if they are related by local unitary transformations. Another fact is that the degree of entanglement of quantum states remains invariant under local unitary transformations. Thus it is an important problem to classify and characterize bipartite and multipartite quantum states under local unitary

transformations.

One approach is based on the theory of invariants. For bipartite mixed states, we present a complete set of invariants such that two density matrices are locally equivalent if and only if all these invariants have the equal values at these density matrices. For the case of multipartite systems a complete set of invariants is also presented for a special class of mixed states. Based on the theory of hyper-determinants, we propose a method of constructing invariants under local unitary transformations, which are independent of the detailed pure state decompositions of a given state.

Based on the generalized Bloch expressions of quantum states, we show that the conditions in the criterion are both sufficient and necessary for multi-qubit systems. In addition, in terms of the tensor decomposition of unitary matrices in composite system, we present a necessary and sufficient criterion for the local unitary equivalence of multipartite states, together with explicit forms of the local unitary operators. The criterion is shown to be still operational for states of eigenvalues with multiplicity no more than 2.