Probabilistic Logic, Probabilistic Regular Expressions, and Constraint Temporal Logic

Thomas Weidner

Abstract

The classic theorems of Büchi and Kleene state the expressive equivalence of finite automata to monadic second order logic and regular expressions, respectively. These fundamental results enjoy applications in nearly every field of theoretical computer science. Around the same time as Büchi and Kleene, Rabin investigated probabilistic finite automata. This equally well established model has applications ranging from natural language processing to probabilistic model checking.

Here, we give probabilistic extensions of Büchi's theorem and Kleene's theorem to the probabilistic setting. We obtain a probabilistic MSO logic by adding an expected value second order quantifier to the logic. In the scope of this quantifier, membership in the quantified set is determined by a Bernoulli process. This approach turned out to be universal, and is applicable for finite and infinite words as well as for finite trees. In order to prove the expressive equivalence of this probabilistic MSO logic to probabilistic automata, we show a Nivat-theorem, which decomposes a recognizable function into a regular language, homomorphisms, and a probability measure. In the case of finite trees, standard top-down probabilistic tree automata are not expressive enough to capture the semantics of all probabilistic MSO sentences. Thus, we consider bottom-up probabilistic tree automata, which form a more powerful model.

For regular expressions, we build upon work of Bollig, Gastin, Monmege, and Zeitoun and extend their probabilistic regular expressions on finite words to the infinite word case, by defining an ω-operator as a certain limit probability. We show the expressive equivalence between these probabilistic ω-regular expressions and probabilistic Muller-automata. To handle Muller-acceptance conditions, we give a new construction from probabilistic regular expressions to Muller-automata. Concerning finite trees, we define probabilistic regular tree expressions using a new iteration operator, called infinity-iteration. Instead of applying tree concatenation arbitrarily often, as with Kleene-iteration, this operator applies it infinitely often (as a limit). This definition utilizes the special use of variables in regular tree expressions, which are not present in the word case. Thus, we obtain probabilistic regular tree expressions with simpler syntax than in the word case. We show that these expressions are expressively equivalent to probabilistic (top-down) tree automata.

On a second track of our research we investigate Constraint LTL over multi dimensional data words with data values from the infinite tree. Such LTL formulas are evaluated over infinite words, where every position possesses several data values from the infinite tree. Within Constraint LTL one can, besides using the usual temporal and Boolean connectives, compare these values from different positions. We show that the model checking problem for this logic is PSPACE-complete via investigating the emptiness of Constraint Büchi automata.