Abstract to thesis

It is a well-known result, that linear representations of finite groups on Hilbert spaces allow for invariant scalar products. Generalizing this property, one calls a group G unitarisable, if every uniformly bounded linear representation ρ of G on some Hilbertspace is similar to a unitary representation. Uniform boundedness her means, that $\sup\{\|\rho(g)\|,\ g \in G\} < \infty$.

In the the 1950's, it was proven, that amenable groups (i.e. groups which allow for a G-invariant finitely additive probability measure on G) are unitarisable. Jaques Dixmier then asked, whether amenable groups are characterized by this property.

Unitarisability of linear representations translates into a fixed-point property for an associated action of G on the cone $\mathcal{P}(H)$ of bounded, positive and invertible operators on H. On this space, there is a metric structure that was introduced by Corach, Porta and Recht. Investigating this and its interplay with the ambient topologies (namely, the restrictions of the weak operator and the norm topologies to $\mathcal{P}(H)$), we will be able to geometrically prove some results about unitarisable groups, which were previously obtained by G. Pisier in a very algebraic way and we can equivalent statements of those theorems in the geometric setup of group actions on $\mathcal{P}(H)$.

After this, we introduce the concept of a GCB-space, which is a generalization of the metric space $\mathcal{P}(H)$ and and a special case of continuous midpoint spaces. In those spaces, we present a general construction of barycenters of finite sets, which depend continuously on the choice of finite sets of same cardinality and is non-expansive.

With the help of such barycenters, we can proof fixed-point properties for groups acting geometrically, amenably and with bounded orbits on GCB-spaces and conclude consequences for actions of groups on $\mathcal{P}(H)$ coming from uniformly bounded representations.

Eventually, we use the barycenter construction to give a generalization of the Ryll-Nardzewski-Theorem to GCB-spaces with additional topological and geometrical properties. The strategy of the proof is very similar to a proof by Glasner.