Kleene-Schützenberger and Büchi Theorems for Weighted Timed Automata

Summary of Dissertation

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In 1994, Alur and Dill [1] introduced *timed automata* for modelling the behaviour of real-time systems. A timed automaton is a finite automaton equipped with a finite set of real valued clocks that measure the time. While being in a state, the values of the clocks increase. When a transition is taken, no time passes, but the values of the clocks may be reset to zero independently of each other. Moreover, one may put timed restrictions on when it is allowed to take a transition by labelling them with constraints on the values of the clocks. Recently [2], timed automata have been extended to *weighted* timed automata. In this model, both the states and transitions are equipped with a natural number that represents the cost for staying in a state or taking a transition, respectively. Since the cost for staying in a state grows linearly with time, this model has many interesting applications, for instance, in operations research and scheduling.

In this thesis, we present a general model of weighted timed automata. The weights for the transitions come from a semiring, which is a well studied mathematical structure appropriate for defining the behaviour of weighted automata. The weights assigned to the states come from a family of functions from the positive reals to the semiring. The behaviour of a weighted timed automaton is a function that maps each *timed word* (i.e., a finite sequence of letters from an alphabet equipped with the time of their occurence) to a coefficient in the semiring, namely its weight. We call such a function a *timed series* and say that it is recognizable if there is a weighted timed automaton with that behaviour. We investigate closure properties of the class of recognizable timed series under the usual operations like sum, Hadamard product and renaming.

The main contribution of this thesis is to give characterizations of recognizable timed series. The first such characterization is an extension of the famous Kleene theorem [4]. We define the class of *rational timed series* using the rational operations sum, Cauchy product, Kleene star iteration and an additional projection operation. We then prove that a timed series is recognizable by a weighted timed automaton if and only if it is rational. Second, we characterize weighted timed automata logically, as it was first done by Büchi [3] for finite automata. We define a weighted timed monadic second order logic and show that a timed series $\mathcal{T}$ is recognizable by a weighted timed automaton if and only if $\mathcal{T}$ is definable by some sentence in a fragment of this logic.

Finally, we investigate the relation between recognizable timed series and recognizable timed languages. In particular, we are interested in the *supports* of recognizable timed series.

We further present interesting results for generalizations of classical decision problems, as e.g. the equivalence problem.

References

[1] R. Alur and D. L. Dill. A theory of timed automata. *Theoretical Computer Science*, 126(2):183–235, 1994.

[2] R. Alur, S. La Torre, and G. J. Pappas. Optimal paths in weighted timed automata. *Theoretical Computer Science*, 318:297–322, 2004.

[3] J. R. Büchi. On a decision method in restricted second order arithmetics. In E. Nagel et al., editor, *Proc. Intern. Congress on Logic, Methodology and Philosophy of Sciences*, pages 1–11, Stanford, 1960. Stanford University Press.

[4] S. C. Kleene. Representation of events in nerve nets and finite automata. In *Automata Studies*, pages 3–42. Princeton University Press, Princeton, NJ, USA, 1956.