Abstract of Dissertation

# Models of Discrete-Time Stochastic Processes and Associated Complexity Measures

Wolfgang Löhr

Many complexity measures are defined as the size of a minimal representation in a specific model class. One such complexity measure, which is important because it is widely applied, is statistical complexity. It is defined for discrete-time, stationary stochastic processes within a theory called computational mechanics. In the thesis, a mathematically rigorous, more general version of this theory is presented, and abstract properties of statistical complexity as a function on the space of processes are investigated. In particular, weak-$*$ lower semi-continuity and concavity are shown, and it is argued that these properties should be shared by all sensible complexity measures. Furthermore, a formula for the ergodic decomposition is obtained. Namely, the complexity of a process is the average complexity of its ergodic components plus the entropy of the mixture. In particular, processes with uncountably many components have infinite complexity.

The same results are also proven for two other complexity measures that are defined by different model classes, namely process dimension and generative complexity. These two quantities, and also the information theoretic complexity measure called excess entropy, are related to statistical complexity, and this relation is discussed in the thesis.

It is also shown that computational mechanics can be reformulated in terms of Frank Knight's prediction process, which is of both conceptual and technical interest. This reformulation on prediction space allows for a unified treatment of different processes and facilitates topological considerations. In particular, it is used in the proof of weak-$*$ lower semi-continuity of statistical complexity and to clarify the relation to process dimension and excess entropy. Continuity of the Markov transition kernel of a discrete version of the prediction process is obtained as a new result.