Bibliographical data

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Rigidity of Hyperbolic Spaces

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Abstract

The idea in Witten's proof of the positive mass theorem is used to give rigidity results of hyperbolic spaces $\mathbb{K}\mathrm{H}^n$ with $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$ as well as of real hyperbolic products. In order to do this a general theory is developed which works for parallel subbundles V of Dirac bundles S. Killing spinors on V are constructed from asymptotic Killing spinors using elliptic theory of the Dirac operator and the non–compact Bochner technique.

With this method it is shown, that a complete, orientable strongly asymptotically hyperbolic manifold with sectional curvature $K \geq -1$ must be isometric to the real hyperbolic space. In this case S is the exterior form bundle and V is given by $\Lambda^1 M \oplus \Lambda^2 M$. The used objects are dual to usual Killing fields.

Another rigidity result of the real hyperbolic space which is based on the existence of twistor spinors on $\mathbb{R}\mathrm{H}^n$ is proven for conformally compact spin manifolds.

Rigidity of the quaternionic hyperbolic space $\mathbb{H}\mathrm{H}^n$ follows from the existence of imaginary quaternionic Killing spinors on the model space: Let (M, g) be a complete strongly asymptotically quaternionic hyperbolic spin manifold of real dimension $4n \geq 12$, then (M, g) is isometric to $\mathbb{H}\mathrm{H}^n$.

Considering the Whitney sum of the spinor bundle of a hyperbolic product with itself, there are special spinors on this bundle. These spinors give the scalar curvature rigidity result of asymptotically real hyperbolic products. But in contrast to the complex and quaternionic hyperbolic case, the proof of this result only needs a weak holonomy assumption.