Convergence of phase-field models and thresholding schemes via the gradient flow structure of multi-phase mean-curvature flow

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Abstract

This thesis is devoted to the rigorous study of approximations for (multi-phase) mean curvature flow and related equations. We establish convergence towards weak solutions of the according geometric evolution equations in the BV-setting of finite perimeter sets. Our proofs are of variational nature in the sense that we use the gradient flow structure of (multi-phase) mean curvature flow. We study two classes of schemes, namely phase-field models and thresholding schemes. The starting point of our investigation is the fact that both, the Allen-Cahn Equation and the thresholding scheme, preserve this gradient flow structure. The Allen-Cahn Equation is a gradient flow itself, while the thresholding scheme is a minimizing movements scheme for an energy that Γ-converges to the total interfacial energy. In both cases we can incorporate external forces or a volume-constraint. In the spirit of the work of Luckhaus and Sturzenhecker (Calc. Var. Partial Differential Equations 3(2):253–271, 1995), our results are *conditional* in the sense that we assume the time-integrated energies to converge to those of the limit. Although this assumption is natural, it is not guaranteed by the a priori estimates at hand.