Stefan Kühn

Hierarchical tensor representation

In this dissertation we present and a new format for the representation of tensors and analyse its properties. The hierarchical format uses a binary tree in order to define a hierarchical structure of nested subspaces in the tensor space of order d. The strorage requirements are $\mathcal{O}(dnr + dr^3)$ where n is determined by the storage requirements in the ansatz spaces and r is a rank parameter determined by the dimensions of the nested subspaces. The hierarchichal representation contains the standard representation like canonical or r-term representation and subspace or Tucker representation. The arithmetical operations that have been developed in this work, including several approximation methods, are based on stable Linear Alebra methods, especially the singular value decomposition (SVD) and the QR decomposition are of importance. The computational complexity is $\mathcal{O}(dnr^2 + dr^4)$. The linear dependence from the order d of the tensor space is important. The approximation methods are one of the key ingredients for the applicability of the new format and we present qualitative and quantitative error estimates. Numerical experiments approve the theoretical results and show some additional, but unexpected positive aspects of the fastest method called FastHOSVD.