Regularity in thin film micromagnetics and some of its generalisation

1 Introduction

Micromagnetism is a paradigm for a pattern forming multi-scale system. The quantity to be predicted is the magnetization $m : \Omega \to S^2$ of a ferromagnetic sample $\Omega \subset \mathbb{R}^3$. There are regions where the magnetisation varies slowly called domains which are $3D$ objects and those regions are seperated by transition layers known as domain walls which are $2D$ objects. There are in general three kinds of domain walls namely Neel walls, bloch walls and cross tie walls. We are mostly interested in the thin film ferromagnetic sample. Neel wall is one of the most dominant character in soft ferromagnetic thin film. The cross-tie wall is observed in relatively thicker films. It is a periodic microstructure of Neel walls and Bloch lines. The asymmetric Bloch wall is observed in even thicker films. It is a stray-field free structure which varies in the thickness direction. The additional small length scales, the film thickness increases the complexity of the pattern. There are different kinds of limiting theories from three dimentional model to two dimentional reduced model depending on different parameter regimes. Mathematically this is a multiscale form of the calculus of variation. If the material exhibits some other property like elastic or electric effect together with magnetism then several other physical situations are possible like magneto-elasticity, electro-magnetism or electro-magneto-elasticity whose mathematical analysis has similarity with micromagnetism.

Let us briefly discuss the main contents of this chapter. We will first present the variational formulation of the micromagnetic model with interpretation of all its terms and constraints. Then we will recall some $3D$ to $2D$ reduction of the model via Γ convergence. Then we will come to our main regularity results. Some partial regularity has been investigated before. We will prove partial $C^{1,\alpha}$ regularity and finally optimal regularity and also a micromagnetic obstacle type problem. The main mathematical results in this chapter is a kind of new weak convergence method in the calculus of variation and an optimal regularity and also a new regularity result for an obstacle like problem. We first show regularity for the minimizer of a penalized problem. Then we show that minimizer of the penalised problem converges strongly to the minimizer of the original problem as the penalization parameter goes to ∞. More precisely, we analyse the regularity of minimum of a dual problem. We recall some partial analysis of regularity of dual problem with a different regularisation. Finally, we return to the original problem and show that there always exists at least one minimum in $C^{0,\beta}$ for some $\beta > 0$. To do so we first established new regularity results for nonconstant gradient constraint problem (this part has been taken from a joint work with S. Muller and J. Andersson) and then apply it to the micromagnetic obstacle problem.

2 The $3D$ Micromagnetic Functional

Micromagnetics is a nonconvex nonlocal variational problem whose local minimizer represents the stable magnetisation patterns of a ferromagnetic body. We will consider the static theory of Landau Lifshitz model of ferromagnetism. This model was introduced by L. Landau and E.M. Lifshitz in 1935 to model the macroscopic theory of ferromagnetism in magnetic material. One of the main theme is the analysis of global minimizer ie ground state magnetisation pattern. The motivation is not that a ferromagnet easily reaches its ground state but rather that the most robust features of the ground state may be shared by all physically accessible local minima.

2.1 The admissibility set

Let $\Omega \subset \mathbb{R}^3$ denotes the ferromagnetic sample. The set of admissible vector fields and potentials respectively $m : \Omega \to \mathbb{R}^3$ and $U : \mathbb{R}^3 \to \mathbb{R}$ satisfies the following constraints

$$|m|^2 = 1$$

and the static Maxwell equation whose variational formulation is

$$\int_{\mathbb{R}^3} \nabla U.\nabla \phi \, dx = \int_{\Omega} m.\nabla \phi \, dx$$

for all $\phi \in C_0^\infty(\mathbb{R}^3)$.

The classical version of the above variational formulation can be written as:

2.2 The Functional

The micromagnetic energy is the sum of four different energy given by

$$I(m) = d^2 \int_{\Omega} |\nabla m|^2 \, dx + Q \int_{\Omega} m_2^2 + m_3^2 \, dx + \int_{\mathbb{R}^3} |\nabla U|^2 \, dx - \int_{\Omega} H_{ext}.m \, dx.$$

2.3 The Reduced variational problem

The reduced variational problem is defined by the set of admissible quantities $m : \omega' \to \mathbb{R}^2$ and $u : \mathbb{R}^3 \to \mathbb{R}$ given by

$$|m'| \leq 1$$

in ω' and

$$\int_{\mathbb{R}^3} \nabla u . \nabla \xi \, dx = \int_{\omega'} m' . \nabla' \xi \, dx'$$

for all $\xi \in C_0^\infty(\mathbb{R}^3)$ and the functional

$$e'(m,u) = \int_{\mathbb{R}^3} |\nabla u|^2 \, dx - 2 \int_{\omega'} h'_{ext} . m'.$$

2.4 The Full variational problem

The full variational problem is defined as follows: Let $d, t \geq 0$ and $\omega = \Omega' \times (0,t)$, $Q = tq$, $H_{ext} = t(h'_{ext}, 0)$. Then the admissible quantities are given by

$$\begin{aligned} |m|^2 &= 1 \\ \int_{\mathbb{R}^3} \nabla U . \nabla \xi \, dx &= \int_{\omega} m . \nabla \xi \, dx \end{aligned}$$

for all $\xi \in C_0^\infty(\mathbb{R}^3)$.

The functional is given by

$$t^2 e(m,U) = d^2 \int_{\omega} |\nabla m|^2 + \int_{\mathbb{R}^3} |\nabla U|^2 \, dx - 2 \int_{\omega} H_{ext} . m \, dx.$$

References

[1] A. Desimone, R. V. Kohn, S. Muller, F. Otto, *A Reduced theory for thin film micromagnetics*, Comm Pure and applied math, 2002, 1408-1460.