Thesis Summary

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Ferromagnetic materials show a large variety of magnetic microstructures. Roughly speaking, one observes regions where the magnetization is almost constant (named magnetic domains) and transition layers separating them (known as domain walls). The theory of micromagnetism explains (some of) these phenomena by (local) minimization of a certain energy functional, the micromagnetic energy E. Various energy contributions with different scaling behavior give rise to multiple length scales and make it a difficult task to solve the micromagnetic minimization problem both analytically and numerically.

Most of the mathematical theory so far has focused on the time-independent case. In my PhD-thesis instead, I study the corresponding evolution equation. This equation is known as Landau-Lifshitz-Gilbert equation (LLG) and can be seen as a hybrid heat and Schrödinger flow for the micromagnetic energy. For the (time-dependent) magnetization $m : \mathbb{R} \times \Omega \to \mathbb{R}^3$ of a (soft) ferromagnetic sample $\Omega \subset \mathbb{R}^3$, it reads as

$$m_t = \alpha\, m \times H_{\text{eff}} - m \times \big(m \times H_{\text{eff}}\big) \quad \text{in } \mathbb{R} \times \Omega \qquad (\alpha \in \mathbb{R})$$

together with homogeneous Neumann boundary conditions and the "saturation constraint" $|m| = 1$. The effective magnetic field H_{eff} is given by $H_{\text{eff}} = \Delta m + H[m] + h_{\text{ext}}$, where $H[m]$ is the so-called stray field and h_{ext} is a given (time-dependent) external magnetic field. My thesis is concerned with the following question:

Question: Do there exist time-periodic solutions for LLG when the external magnetic field is periodic in time?

Due to the complexity of LLG, one can not expect to obtain a universally valid answer to that question. It is advantageous to restrict oneself to certain parameter regimes as it is done for the (time-independent) micromagnetic minimization problem. In my thesis, two different situations are treated. More precisely, I study LLG for soft and small particles in the first part (Chapters 1 and 2), and in the second part (Chapter 3), I investigate time-periodic Néel wall motions.

In Chapter 1, I construct time-periodic solutions for LLG in the regime of soft and small ferromagnetic particles. 'Soft' refers to the case when no energetically preferred directions (so-called easy axis) exist, and 'small' means that $|\Omega| \ll 1$, where $|\Omega|$ denotes the volume of Ω. Assuming that, roughly speaking, the length of Ω is greater than its height and its width, I prove the existence of time-periodic solutions for LLG with respect to time-periodic external magnetic fields h_{ext}. The proof involves a perturbation argument as well as the theory of sectorial operators. The idea is to consider the minimizers of the micromagnetic energy functional as stationary solutions for LLG with $h_{\text{ext}} = 0$. By introducing a new parameter for the amplitude of h_{ext}, one can apply the implicit function theorem on an infinite-dimensional manifold to construct time-periodic solutions for nonvanishing time-periodic external magnetic fields with small amplitudes. In this procedure, the restriction on the shape of Ω is needed to guarantee the invertibility of the corresponding linearization. To the best of my knowledge, the obatined result is the first existence result concerning time-periodic solutions for LLG.

The regularity properties of minimizers of the micromagnetic energy functional are another crucial ingredient for the above-mentioned existence result. The smallness requirement on Ω guarantees that minimizers of E belong to the function space $H^2(\Omega, \mathbb{R}^3) \cap C^{1,\gamma}(\overline{\Omega}, \mathbb{R}^3)$ and satisfy a certain L^∞-estimate for the gradient, provided the boundary of Ω is sufficiently smooth. The proof of this statement is the major topic of my second chapter. There, I treat minimizers of E as almost minimizers of the Dirichlet energy and adapt methods from the regularity theory for minimizing harmonic maps into spheres. With help of a special coordinate system, it is possible to analyze interior and boundary regularity at the same time. The thereby obtained small-energy-regularity theorem combined with a covering argument implies the required regularity result for the magnetization of a small particle.

In the second part, I study time-periodic Néel wall motions. The Néel wall, modeled by $m : \mathbb{R} \to S^2$, separates two domains with magnetizations of opposite directions by a planar 180°-rotation. It is observed in soft and thin films. Characteristic properties of the Néel wall are the very long logarithmic tail of the transition profile and logarithmic energy scaling. For a certain parameter regime, I prove the existence of time-periodic solutions for LLG close to the static Néel wall profile. The used methods include the implicit function theorem and the spectral analysis of the corresponding linearized problem. It turns out that the linearization possesses a nontrivial kernel, and with the help of a spectral gap estimate it is shown that the kernel is actually one-dimensional. The resulting properties of the analytic semigroup generated by the linearization allow the construction of time-periodic solutions close to the static Néel wall for certain time-periodic external magnetic fields.