THESIS ABSTRACT

THE MOVING CONTACT LINE IN VISCOUS THIN FILMS: A SINGULAR FREE BOUNDARY PROBLEM

MANUEL VICTOR GNANN

Date of submission: March 24, 2014
Date of the defense: July 23, 2014

We investigate a free boundary problem for the thin-film equation, which is a fourth-order degenerate parabolic equation that models the flow of a viscous thin film on a flat (one-dimensional) solid. Here, we are interested in the dynamics of the free boundary, i.e. the contact line (triple junction) separating the phases liquid, gas, and solid. Assuming a no-slip condition at the liquid-solid interface, this problem exhibits the well-known no-slip paradox, that is, the contact line can only move by inserting an infinite amount of dissipative energy into the system. Hence, we assume a linear Navier-slip condition at the liquid-solid interface leading to a quadratic mobility in the equation and a regularization at the free boundary. Furthermore, we assume a zero contact-angle condition at the triple junction, commonly referred to as 'complete wetting regime'.

For this problem, a well-developed existence theory of weak solutions is available. However, the question of well-posedness has only recently been addressed by Giacomelli, John, Knüpfer, and Otto. This question is related to the understanding of the regularity at the contact line: Sufficient regularity allows to obtain a solution by a contraction argument and not only by weak compactness.

The thesis is structured as follows: First, we discuss the regularity of source-type solutions for a specific class of thin-film equations, in particular containing the case of linear Navier slip. Factoring off the leading-order traveling wave, we show that solutions are not smooth up to the contact line, but are analytic in two variables that are (non-integer) powers of the distance to the boundary.

Subsequently, we investigate the general fourth-order problem with quadratic mobility and prove global well-posedness for small perturbations of traveling waves. The main ingredients for the proof are maximal regularity estimates in weighted L^2-spaces for the linearized evolution, after suitably subtracting the leading-order singular expansion at the boundary.

Next, we study small perturbations of source-type solutions for the thin-film equation with linear mobility (coming from Darcy dynamics) and prove long-time existence and uniqueness. From the work of Carrillo and Toscani it is known that these solutions are global attractors for rather general initial data (compactly supported or with finite second moment). Since we exclude spatial translations in our reference frame, the rate of convergence is faster than the one obtained by Carrillo and Toscani.

Finally, we are concerned with a more general version of the thin-film equation with inhomogeneous mobility, modeling films with heights that are larger than the slip length. While at the contact line the solutions again exhibit analyticity in two variables being powers of the distance to the free boundary (asymptotic regime I), for film heights being large compared to the slip length, we observe another asymptotic regime that is known as Tanner's law (II). Here we show that – within our framework – the precise physics at the contact line (I) do not affect the shape of the film far away from it (II) – apart from trivial shift and scaling transformations and a higher-order correction.