

Global solutions and finite time blowup in a two species model for chemotaxis

Abstract

Keywords: Chemotaxis, blowup, symmetric solutions, drift diffusion system.

Mathematics Subject Classification: 35M10, 35Q80, 35K50, 92B05, 92D99

We analyzed the two chemotactic species model

$$\begin{aligned}\partial_t u_1 &= \Delta u_1 - \chi_1 \nabla \cdot (u_1 \nabla v), & \text{in } \Omega, t > 0, \\ \partial_t u_2 &= \mu_2 \Delta u_2 - \chi_2 \nabla \cdot (u_2 \nabla v), & \text{in } \Omega, t > 0, \\ 0 &= \gamma \Delta v + \alpha_1 u_1 + \alpha_2 u_2 - 1, & \text{in } \Omega, t > 0,\end{aligned}$$

in radial symmetric situation in order to see if chemotaxis can separate the two species. From our results so far this does not seem to be the case. There is simultaneous blow-up for both chemotactic species. Specifically it is proved that if there is blow-up in one chemotactic species then there is blow-up in the other one. We give conditions for local and global existence. The existence of blow-up in this system is proved too.

In the drift diffusion system

$$\begin{aligned}\partial_t n - \Delta n + \nabla \cdot (n \nabla \psi) &= 0, & t > 0, & x \in \Omega \\ \partial_t p - \Delta p - \nabla \cdot (p \nabla \psi) &= 0 & t > 0, & x \in \Omega \\ -\Delta \psi &= -(p - n). & x \in \Omega \\ n(0, x) = n_0(x) > 0, & p(0, x) > 0, & x \in \Omega,\end{aligned}$$

it is proved the existence of blowup. Conditions for non-simultaneous blow-up are given.