Generalizations and Interpretations of Incipient Infinite Cluster Measure on Planar Lattices and Slabs

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For critical planar percolation, although there is no infinite open component, there exists giant clusters on every macroscopic scale. It is reasonable to believe that local patterns around vertices of large spanning clusters appear with frequencies given by a probability measure on occupancy configurations. This measure would inherit properties of critical percolation, but would be supported on configurations with an infinite open cluster at the origin. Kesten gave a first mathematically rigorous construction of such a measure (which we refer to as Incipient Infinite Cluster measure) in two distinct ways and showed that they coincide. Later, Járai proved that Kesten's Incipient Infinite Cluster (IIC) measure indeed describes frequency of local patterns around a typical point in large crossing clusters.

Our work in generalizing and interpreting Kesten's IIC measure is split into two parts. Firstly we generalize Járai's result for describing configurations around a typical point taken from certain exceptional sets related to open crossings of boxes, namely – backbone, lowest crossing, and set of pivotals. We show that the local picture around them are given by certain multiple-arm IIC measures, whose existence were shown earlier by Damron and Sapozhnikov.

Secondly, we prove the existence of Kesten's IIC measure for two-dimensional slabs. In fact, we prove that, for Bernoulli percolation on any infinite connected bounded degrees graph satisfying two assumptions, which are (A1) uniqueness of the infinite open cluster, and (A2) quasi-multiplicativity of crossing probabilities, both interpretations of Kesten's IIC measure will exist and coincide. Assumption (A1) is true for any sufficiently regular amenable graph including slabs and d-dimensional integer lattice. Thus, we prove this works for slabs by verifying (A2) quasi-multiplicativity for slabs. This is the first instance where both definitions of Kesten's IIC measure is shown to exist and coincide for non-planar graphs. (We expect that quasi-multiplicativity holds for low dimensional integer lattices too, thus our method is a promising first step towards the construction of Kesten's IIC on such lattices.) One of the constituents of this proof worth mentioning is a generalization of a vital tool for critical planar percolation - namely, Russo Seymour Welsh theorem. Finally, inspired again by Járai, we interpret Kesten's IIC measure in slabs as local picture around typical points of crossing collection.